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Quantum Effects near the Singularity in a General Cosmological Scenario

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Joshi and Joshi obtained a differential equation for quantum uncertainty under some very general conditions and inferred the divergence of the uncertainty on the basis of an approximate solution. The present note exactly solves the equation and confirms the divergence.

1. INTRODUCTION

In a recent paper Joshi and Joshi (1987) discussed the occurrence of a singularity in a general cosmological scenario with due consideration of quantum effects and obtained the following differential equation:

$$\frac{d^2\chi}{dt^2}+\frac{1}{t}\frac{d\chi}{dt}=\frac{2\omega}{K_1^2 t} \tag{1}$$

where χ, the quantum uncertainty, which indicates the possible departure from the classically singular state, and $\omega = \langle p^2 \rangle$, the corresponding uncertainty in the momentum p, are connected by the relation

$$\chi\omega \geq \tfrac{1}{4} \tag{2}$$

For $\chi\omega = \frac{1}{4}$ the differential equation (1) is reduced to

$$t^2\frac{d^2\chi}{dt^2}+t\frac{d\chi}{dt}=\frac{1}{2K_1^2\chi} \tag{3}$$

Joshi and Joshi obtained an approximate solution of equation (3) which led to some significant results. In the present note I verify those results by obtaining the exact solution of equation (3).

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2. EXACT SOLUTION AND ITS STUDY

Equation (3) can be rewritten as

$$\frac{d^2\chi}{du^2}=\frac{1}{2K_1^2\chi} \tag{4}$$

where

$$u=\ln t \tag{5}$$

On integration, equation (4) gives

$$\left(\frac{d\chi}{du}\right)^2=l+\frac{1}{K_1^2}\ln|\chi| \tag{6}$$

where l is a constant.

From (6), using (5),

$$\pm(m+\ln|t|)=\int\frac{d\chi}{[l+(1/K_1^2)\ln\chi]^{1/2}} \tag{7}$$

where m is a constant.

Now substituting

$$\chi=\exp(y^2-lK_1^2) \tag{8}$$

in (7), we obtain

$$m+\ln t=\pm 2K_1\exp(-lK_1^2)\int\exp(y^2)\,dy \tag{9}$$

Now as $t\to 0$, the left-hand side of equation (9) tends to ∞, which means that the right-hand side of equation (9) tends to ∞. This is possible only when $y\to\infty$. Now, as $y\to\infty$, equation (8) shows that the quantum uncertainty χ tends to ∞ for $t\to 0$, which confirms the result obtained by Joshi and Joshi.

3. CONCLUSION

In summary, the exact solution of equation (3) is given by equation (9), where χ is given by equation (8). This exact solution confirms the inference of the divergence of χ made in Joshi and Joshi's (1987) approximate study.

REFERENCE

Joshi, Pankaj S., and Joshi, Sonal S. (1987). *Physics Letters A*, **121**(7), 336.